Signal Detection Using Difference Sets

W. R. Smith and W. P. du Plessis

Email: wadersmith89@gmail.com, wduplessis@ieee.org

Introduction

Continued developments in fields such as wideband communications and electronic warfare (EW) have led to ever increasing signal bandwidths, such that sampling at the Nyquist rate is becoming more and more difficult, and produces a large number of samples required for perfect reconstruction of signals. However, there are several cases where perfect reconstruction is not required, and as such allow for sampling below the Nyquist rate. One such case is the detection of signals in noise; the goal of signal detection is to determine whether or not a particular signal is present and obtain a reasonable estimate of it, rather than to perfectly reconstruct it.

Discarding large numbers of samples taken in the time domain can be viewed as similar to discarding a large number of antenna elements in a filled antenna array, a technique known as massive thinning. Historically, cut-and-try random placement of elements has proven as effective as most deterministic approaches, but this typically results in a dramatic loss of sidelobe control [1]. However, deterministic element placement using difference sets has been shown to produce power patterns with well-controlled sidelobes, and a reduction in peak sidelobe level when compared to random element placement [1], [2]. Based on these results, it would appear that difference sets could be applicable to the problem of deterministically discarding time domain samples prior to signal detection, while maintaining adequate detection performance.

Difference Sets

Difference sets are a concept from the branch of mathematics known as combinatorics. A (V, K, Λ) difference set

$$D=\{d_k \in [0, V-1]; k=0,\ldots,K-1\} \quad (1)$$

is a subset of a group G of order V such that the set of differences

$$M=\{m_j=(d_h-d_l); d_h \neq d_l; j=0,\ldots,K\times(K-1)-1\} \quad (2)$$

contains every nonzero element of G each exactly Λ times. A difference set D can be used to construct a binary sequence

$$A_V=\{a_0, a_1, \ldots, a_{V-1}\} \quad (3)$$

where a_j = 1 if j is in D and a_j = 0 if j is not. In antenna array thinning, this binary sequence dictates the location of elements in the array; a one indicates that an element should be placed at a location, and a zero indicates that it should not. By periodically repeating A_V to create an infinite sequence A_I, the autocorrelation function can then be defined as

$$C_I(\tau)=\sum_{n=0}^{V-1} a_n \times a_{n+\tau} \,. \quad (4)$$

If and only if A_I is formed from a difference set, the autocorrelation function reduces to

$$C_I(\tau)=\begin{cases} K, & \text{if } \tau \bmod V=0 \\ \Lambda, & \text{otherwise.} \end{cases} \quad (5)$$

which is two-valued. It has been shown that if the location of the antenna elements is dictated by an infinite sequence formed using a difference set, the resulting array power pattern has all of the peaks of the sidelobes constrained to less than *1/K* times the main beam; if the infinite sequence is truncated to a single sequence period, these levels remain exactly the same, but only half of the sample points of the power pattern are tied to them [1]. This effect can be seen in Figure 1 for the difference set (127, 63, 31).

Difference sets have another very useful property; they are cyclic. Given a (V, K, Λ) difference set D, the set

$$D'=\{d_0+s, d_1+s, \ldots, d_{k-1}+s\}=D+s, \quad (6)$$

where each element is taken modulo V, is also a (V, K, Λ) difference set; D' is called a cyclic shift of D. Since D' has the same parameters as D, this means that it will display exactly the same sidelobe performance as D, for all possible cyclic shifts.

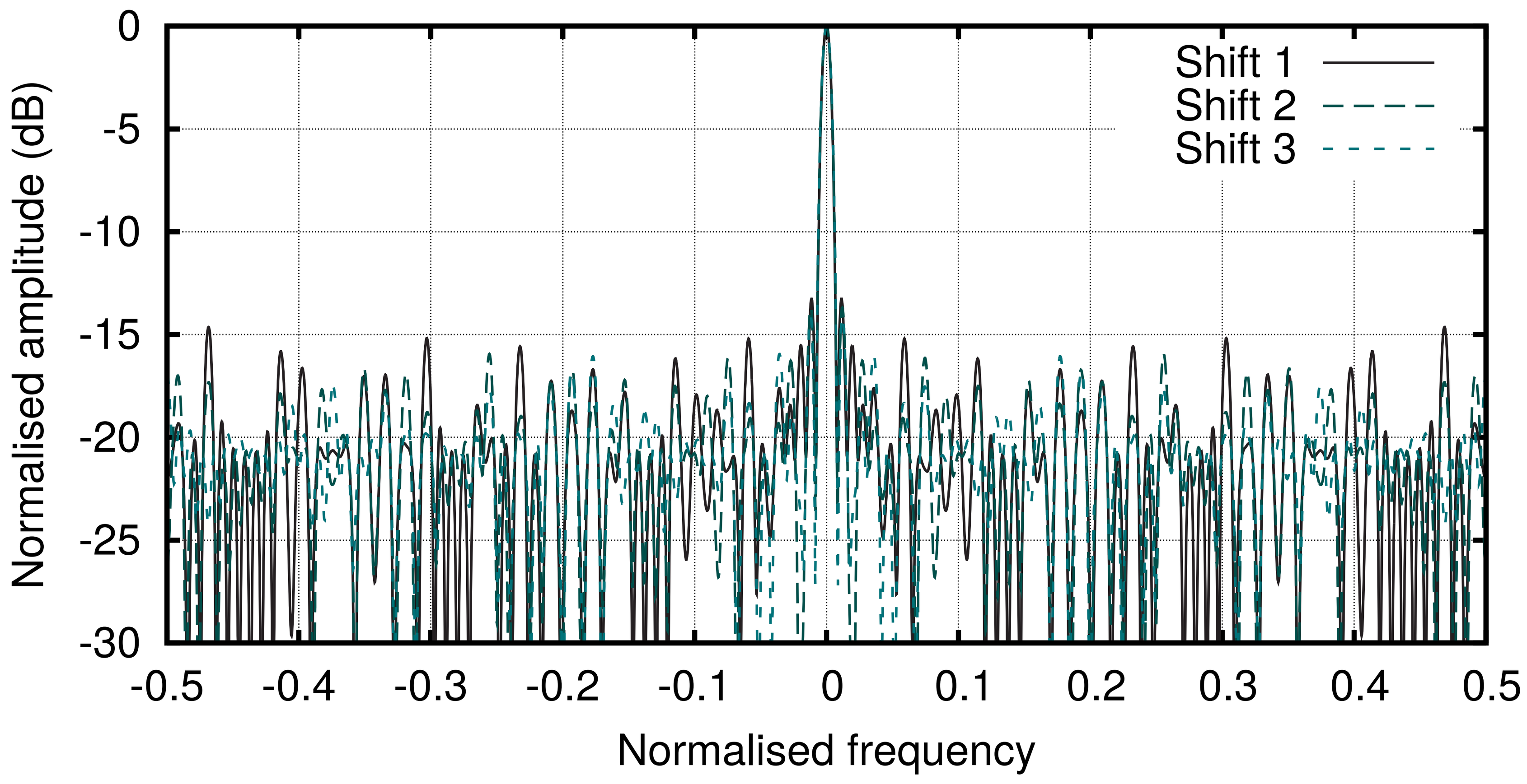

Figure 1. The sidelobes of a binary sequence formed from the difference set (127, 63, 31).

This cyclic property is of particular importance for the application of difference sets to signal detection. Probability of detection can be improved by processing multiple time samples together within a finite window. Reducing the number of samples available in a window should result in a reduction in the probability of detection [3]. However, the cyclic property means that there is always perfect overlap with the signal, i.e. the full signal is considered, not just the portion which happens to appear in a given window, which may help to compensate for the reduction in the number of samples. Additionally, the sidelobe performance should not change significantly as the window effectively shifts in time.

Conclusion

Difference sets show promise for application in signal detection to effectively reduce the number of samples required to achieve adequate detection performance. The sidelobe performance that has been shown for massively thinned antenna arrays should be applicable to this work, since the mathematical basis is very similar, and the deterministic nature of the sets allows for predictions about the detection performance to be made a-priori.

One possible application of this work is the detection of cellphone signals in order to identify and prevent poaching activities. Electric unmanned aerial systems (UASs) are currently being developed to assist in anti-poaching operations; such platforms survey large areas of land, but due to their limited onboard processing, they often transmit sampled data back to a base station for analysis [4]. A reduction in the number of samples that require transmission would result in a reduction in the energy required to transmit them, which could lead to an increase in the range and endurance of the system.

References

[1] D. G. Leeper, “Isophoric arrays - massively thinned phased arrays with well-controlled sidelobes,” *IEEE Trans. Antennas Propag.*, vol. 47, no. 12, pp. 1825–1835, 1999.

[2] G. Oliveri, M. Donelli, and A. Massa, “Linear array thinning exploiting almost difference sets,” *IEEE Trans. Antennas Propag.*, vol. 57, no. 12, pp. 3800–3812, 2009.

[3] J. B. Y. Tsui, *Digital Techniques for Wideband Receivers*, 3rd ed., SciTech Publishing, 2015.

[4] J. N. Koster, A. Buysse, L. Smith, J. Huyssen, J. Hotchkiss, J. Malangoni, and J. Schneider, “AREND: A sensor aircraft to support wildlife rangers,” in *57th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, 2016.

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Engineering, Built Environment and Information Technology

Fakulteit Ingenieurswese, Bou-omgewing en Inligtingtegnologie / Lefapha la Boetšenere, Tikologo ya Kago le Theknolotši ya Tshedimošo

Electronic Defence Research

Contact: Prof. Warren du Plessis (wduplessis@ieee.org)

www.up.ac.za/cedr

www.up.ac.za/eece